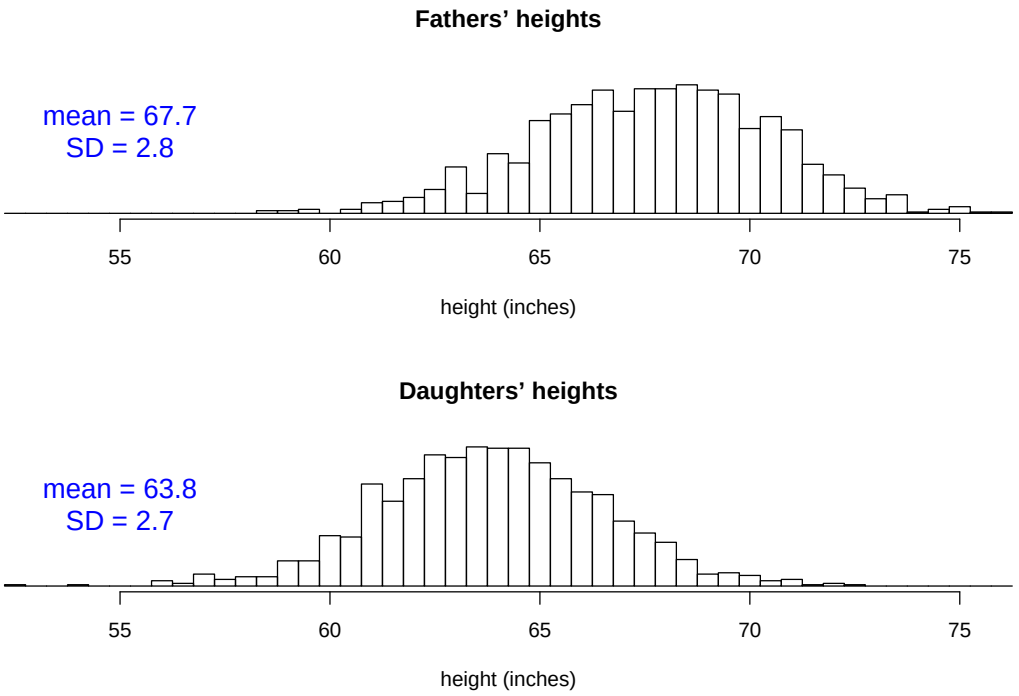
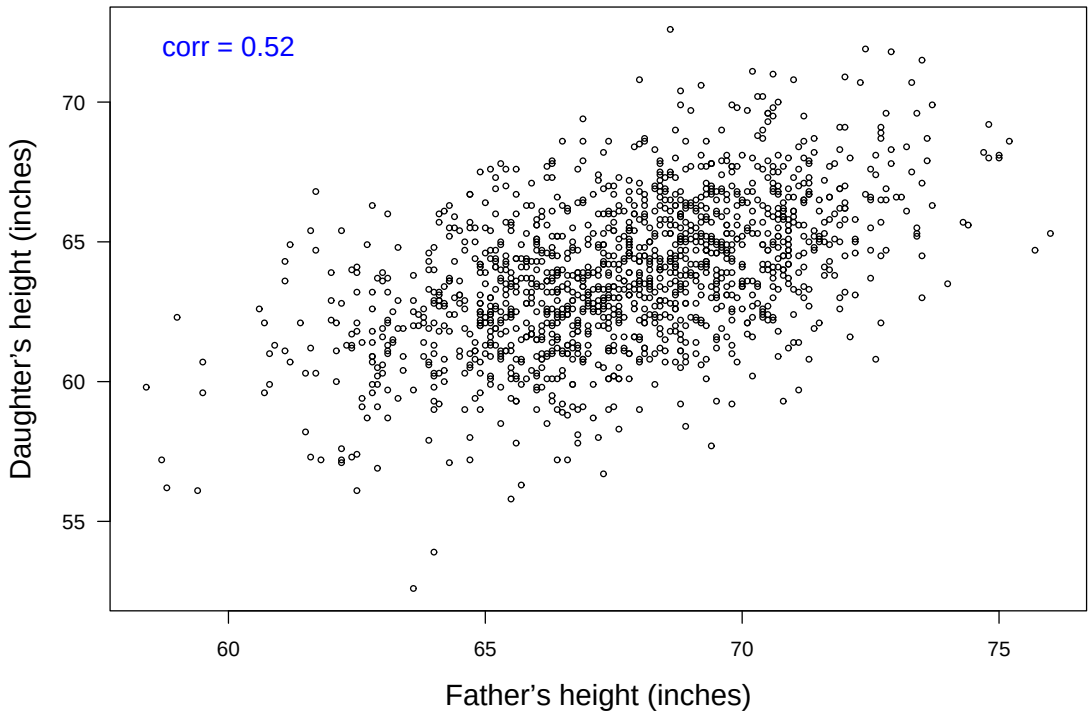
Fathers' and daughters' heights



Pearson and Lee (1906) Biometrika 2:357-462

1376 pairs

Fathers' and daughters' heights



Pearson and Lee (1906) Biometrika 2:357-462

1376 pairs

Covariance and correlation

Let X and Y be random variables with

μ_X = E(X), μ_Y = E(Y), σ_X = SD(X), σ_Y = SD(Y)

For example, sample a father/daughter pair and let

X = the father's height and Y = the daughter's height.

Covariance

$\text{cov(X,Y)} = \text{E}\{(\text{X} - \mu_X)\ (\text{Y} - \mu_Y)\}$

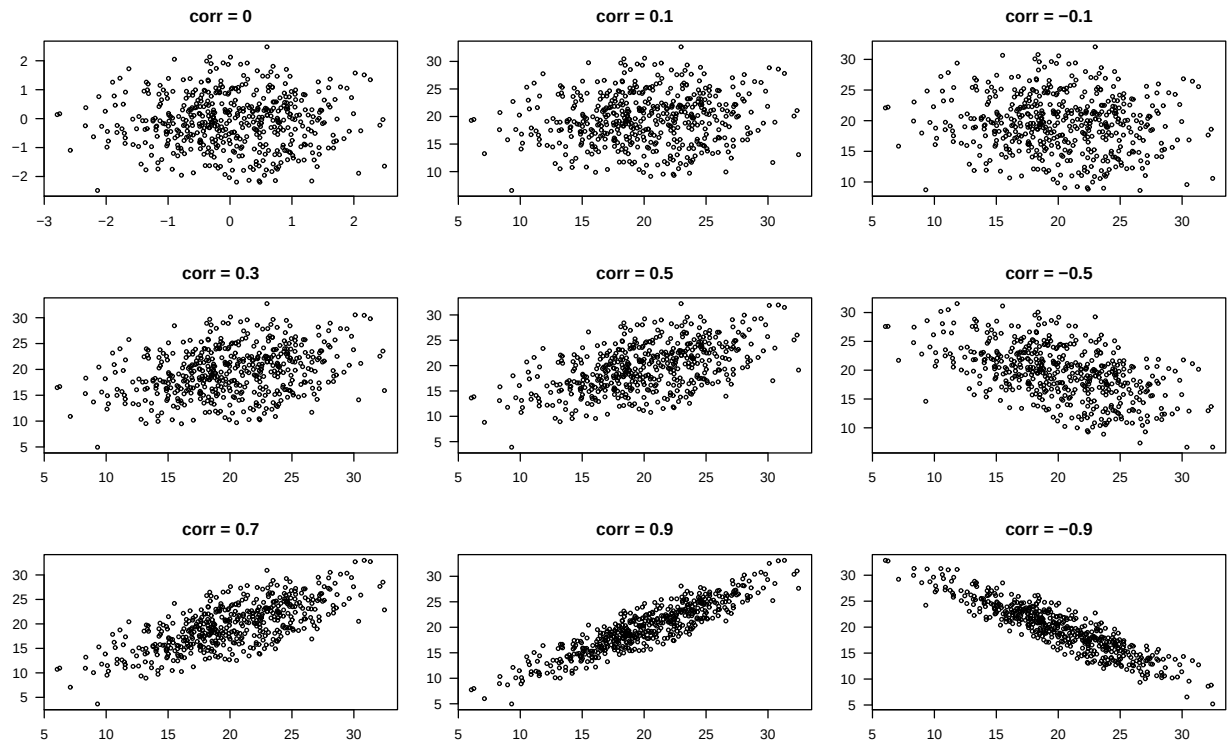
cov(X,Y) can be any real number.

Correlation

$$\text{cor}(\text{X}, \text{Y}) = \frac{\text{cov}(\text{X}, \text{Y})}{\sigma_X \sigma_Y}$$

$$-1 \leq \text{cor}(\text{X}, \text{Y}) \leq 1$$

Examples



Estimated correlation

Consider n pairs of data: $(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_n, y_n)$

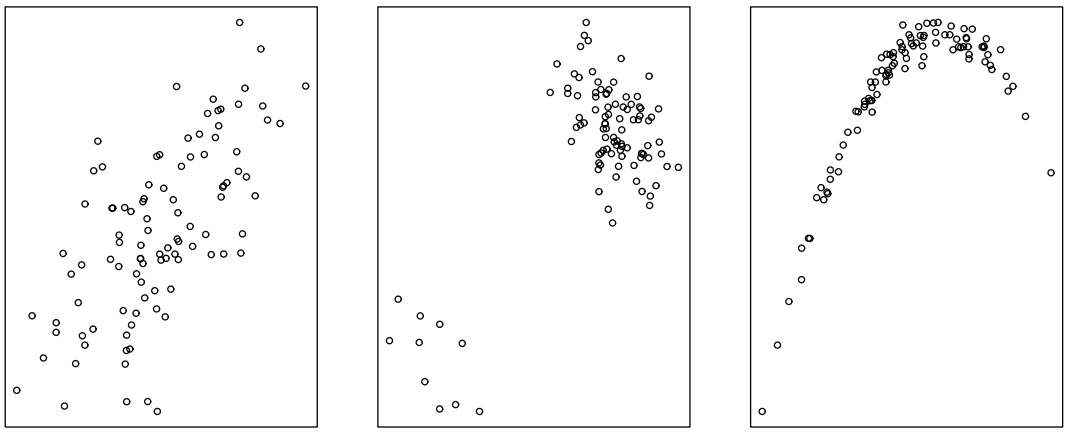
We consider these as independent draws from some bivariate distribution.

We estimate the correlation in the underlying distribution by:

$$r = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2 \ \sum_i (y_i - \bar{y})^2}}$$

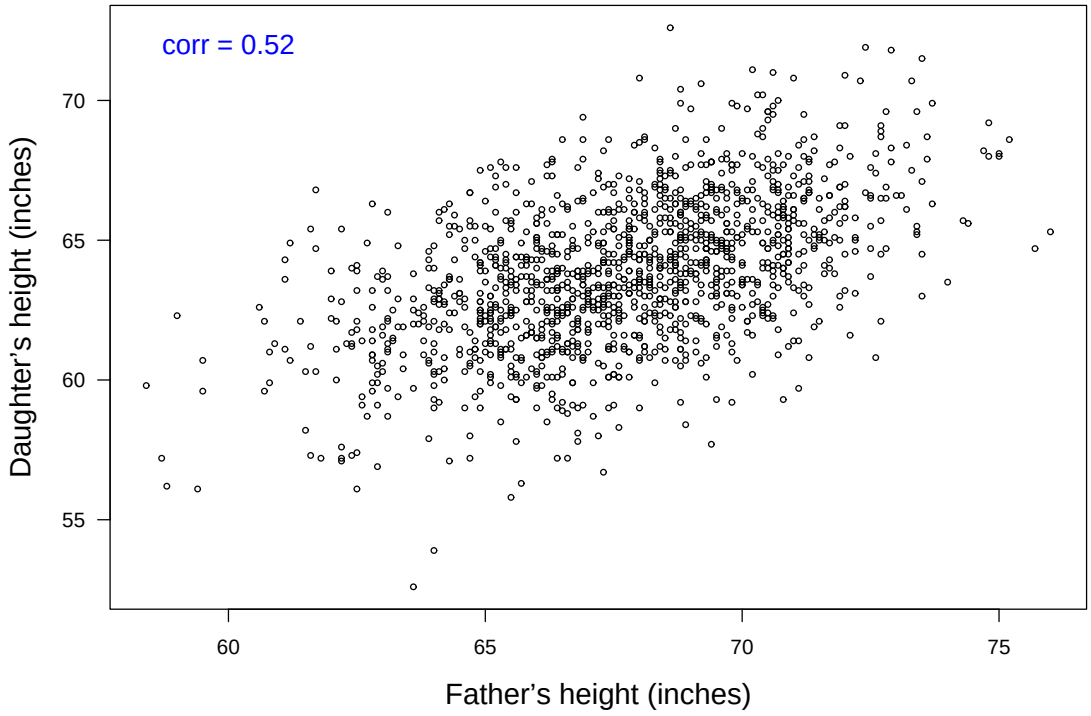
This is sometimes called the correlation coefficient.

Correlation measures linear association

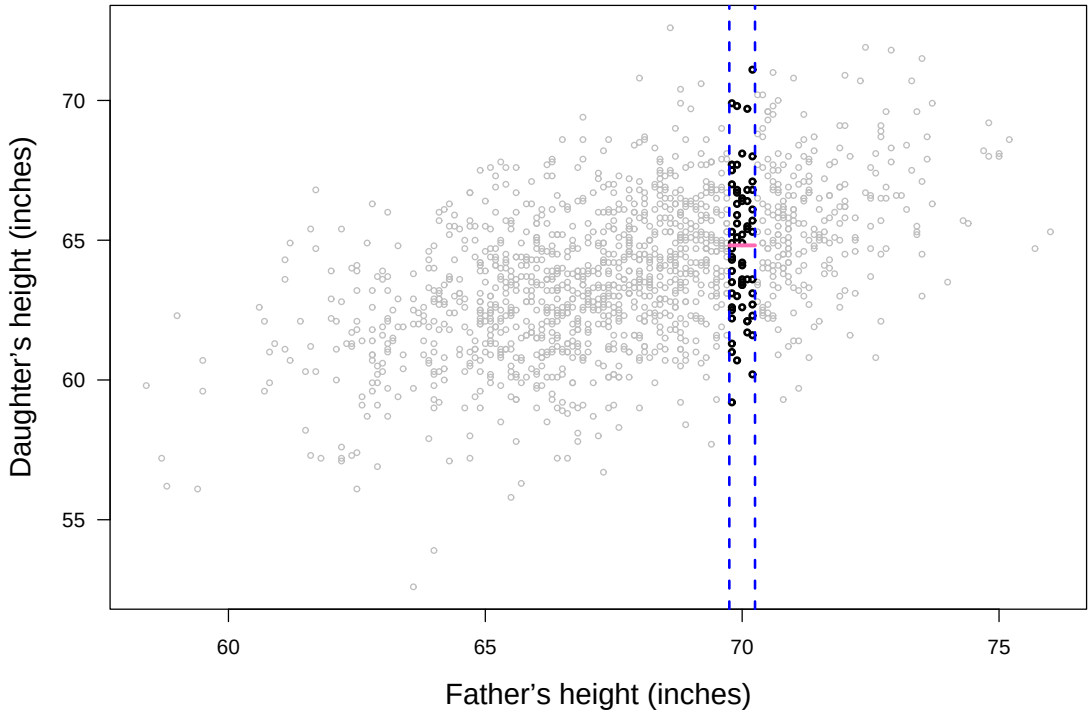


All three plots have correlation ≈ 0.7!

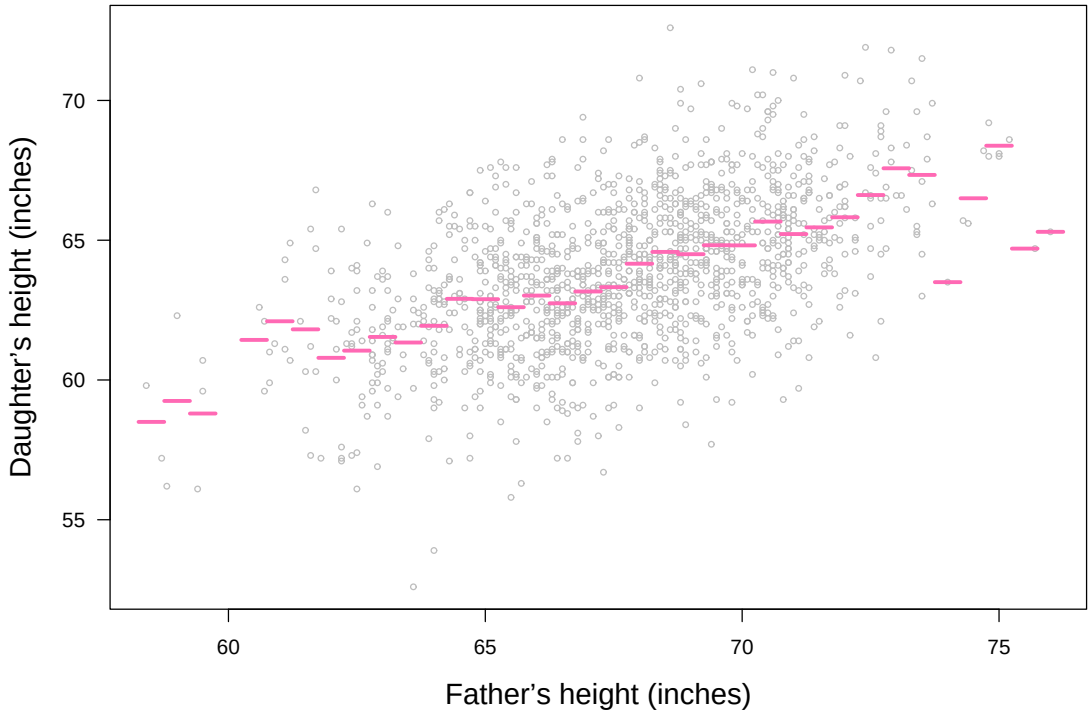
Fathers' and daughters' heights



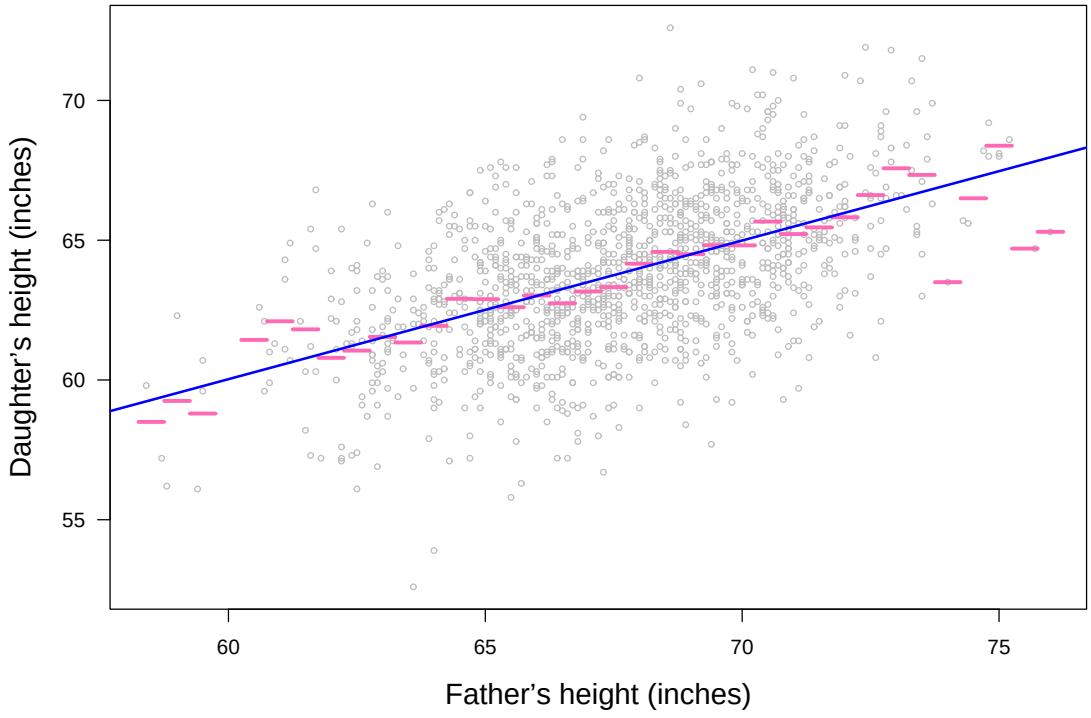
Linear regression



Linear regression

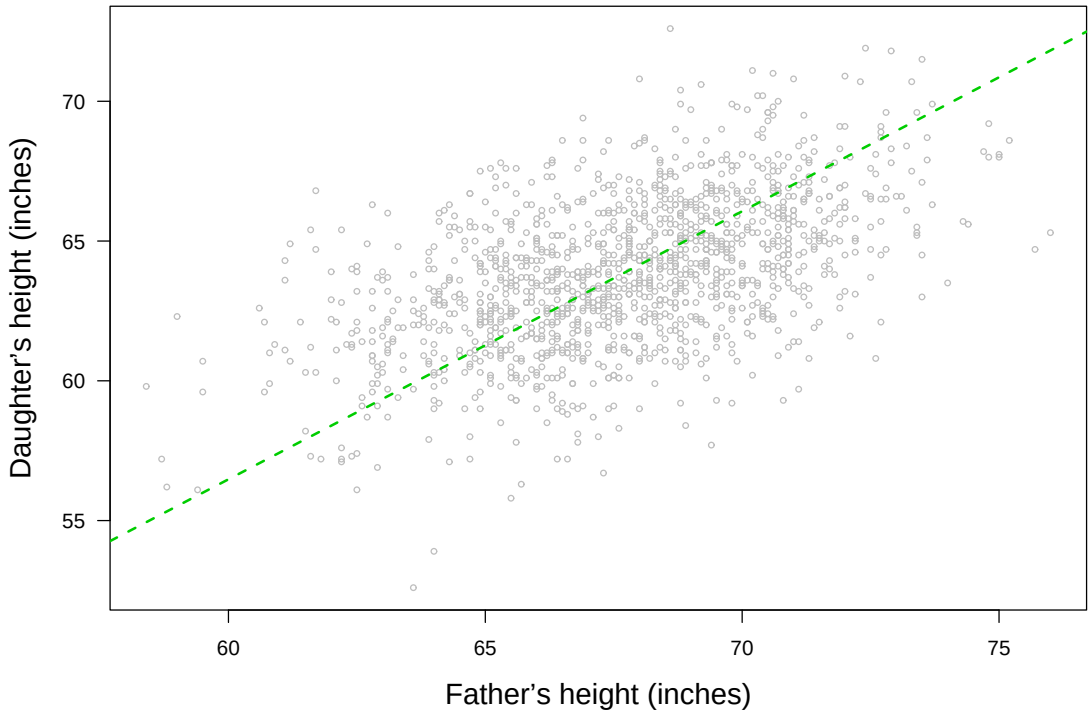


Regression line



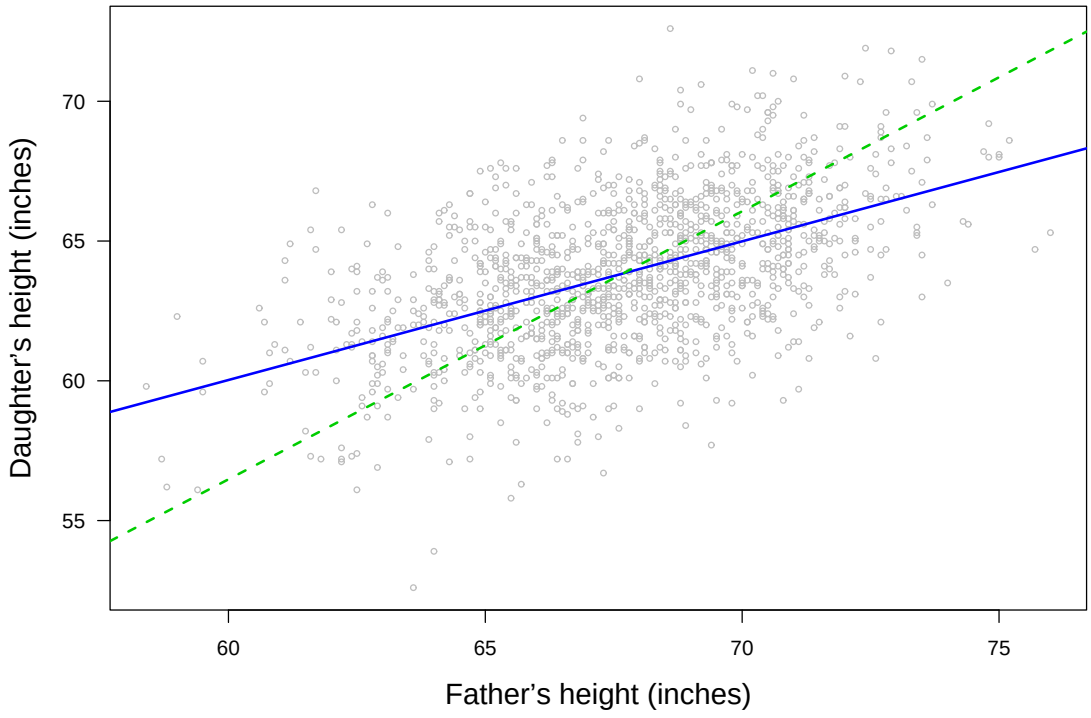
Slope = r × SD(Y) / SD(X)

SD line



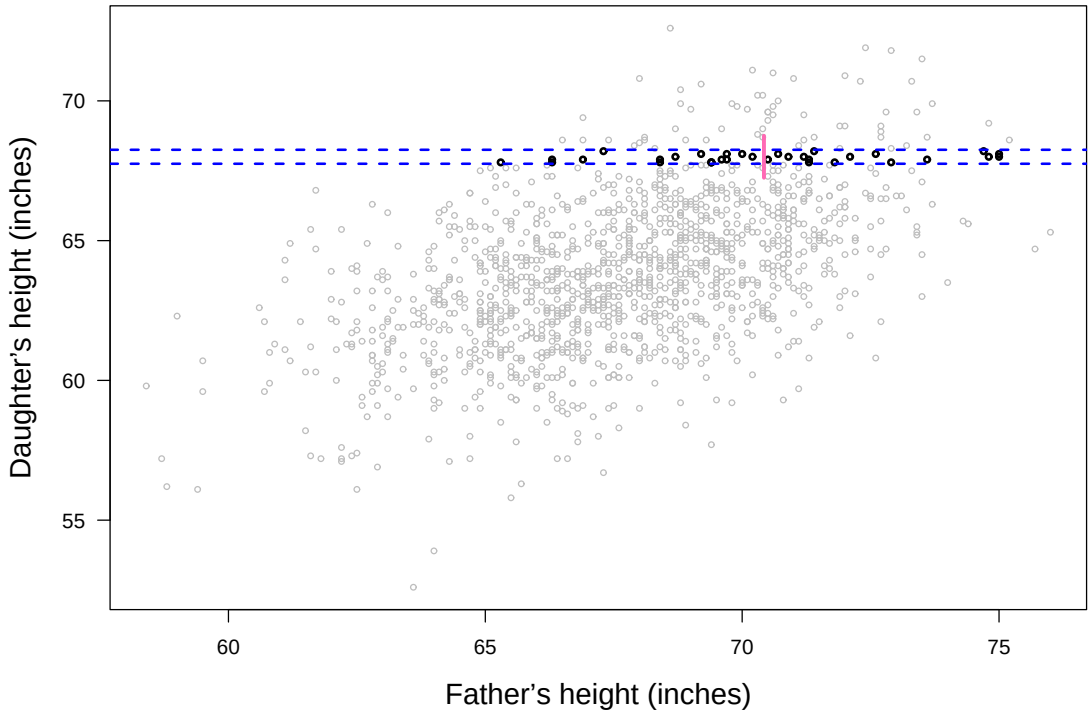
Slope = SD(Y) / SD(X)

SD line vs regression line

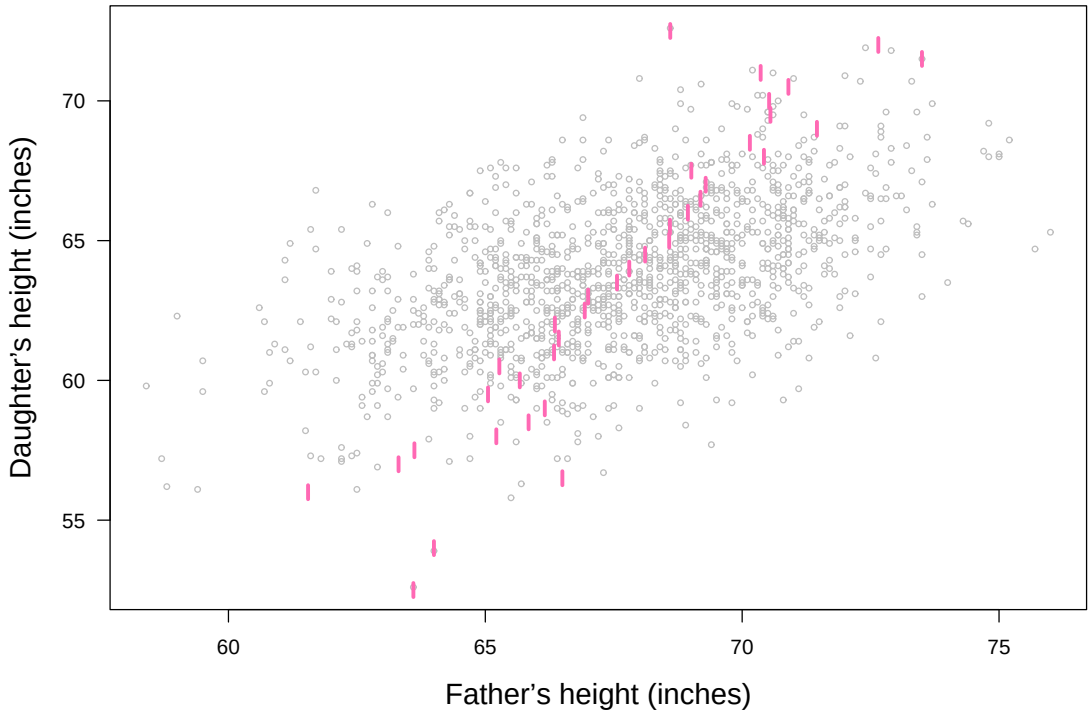


Both lines go through the point ($\bar{X}$, $\bar{Y}$).

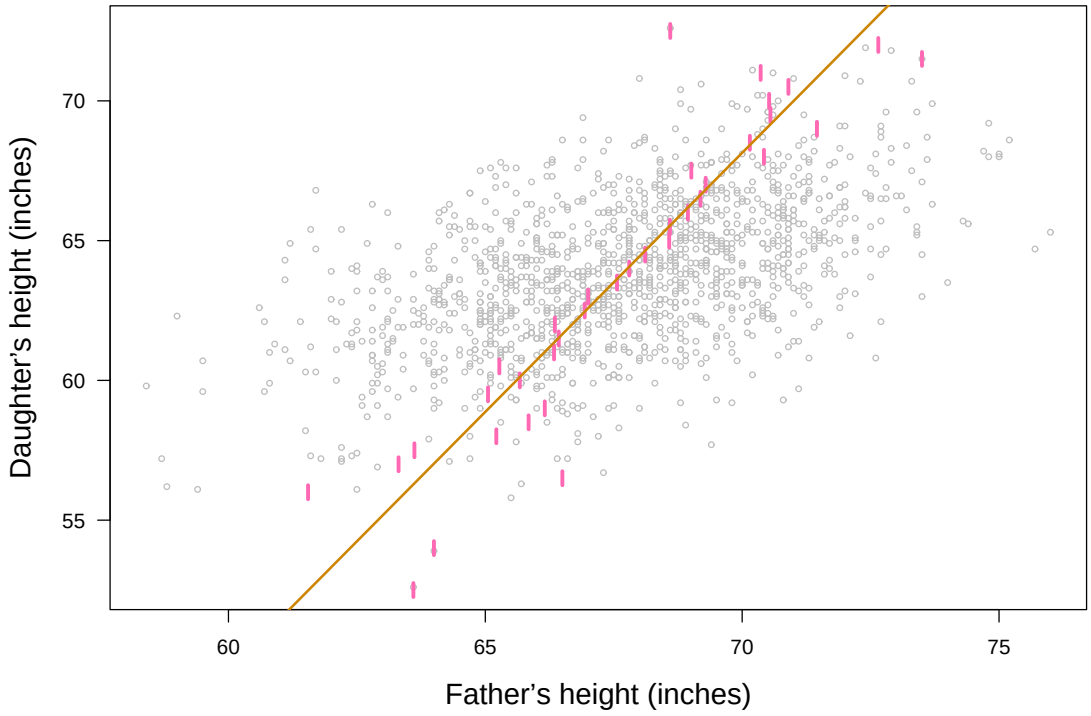
Predicting father's ht from daughter's ht



Predicting father's ht from daughter's ht



Predicting father's ht from daughter's ht



There are two regression lines!